NSWCDD/TR-92/241

(2)

RESISTIVE GRID KERNEL ESTIMATOR (RGKE)

BY WENDY POSTON GEORGE ROGERS
CAREY PRIEBE JEFFREY SOLKA
STRATEGIC SYSTEMS DEPARTMENT

JUNE 1992

NAVAL SURFACE WARFARE CENTER
DAHLGREN DIVISION
Dahlgren, Virginia 22448-5000

NSWCDD/TR-92/241

RESISTIVE GRID KERNEL ESTIMATOR (RGKE)

BY WENDY POSTON GEORGE ROGERS
CAREY PRIEBE JEFFREY SOLKA
STRATEGIC SYSTEMS DEPARTMENT

JUNE 1992

Approved for public release; distribution is unlimited.

NAVAL SURFACE WARFARE CENTER
DAHLGREN DIVISION
Dahlgren, Virginia 22448-5000

FOREWORD

The problem of estimating a probability density function is addressed in this report. The algorithm presented here is the kernel estimation method using a resistive grid network suitable for hardware implementation. Results show that the linear Resistive Grid Kernel Estimator (RGKE) yields estimates that are comparable to those formed using a Gaussian kernel. It will also be shown that incorporating the inherent nonlinearities of the RGKE allows the detection of discontinuities in the density function.

This work has been supported by a number of sponsors and has been conducted in the Space and Ocean Geodesy Branch.

This report has been reviewed by Dr. Richard A. Lorey, Head, Space and Ocean Geodesy Branch and James L. Sloop, Head, Space and Surface Systems Division.

Approved by:

R. L. Schmidt

R. L. SCHMIDT, Head
Strategic Systems Department

ABSTRACT

The ability to estimate a probability density function from random data has applications in discriminant analysis and pattern recognition problems. A resistive grid kernel estimator (RGKE) is described that is suitable for hardware implementation. The one-dimensional linear RGKE is compared to a kernel estimate using Gaussian kernels, and simulations are presented using both continuous and quantized data. The nonlinear form of the RGKE is shown to have desirable properties, such as the ability to detect discontinuities in the density function.

CONTENTS

ILLUSTRATIONS

INTRODUCTION

In this report, the problem of estimating the probability density function f(x) of a given sample of n real observations $X_1, \ldots, X_n$ will be considered. The approach studied here is the kernel estimator from Silverman[1] using a kernel that can be implemented by a resistive grid network.[2] A discussion of kernel estimates and their application to neural networks can be found in Specht,[3] and a review of research concerning pattern classification is presented in Lippmann.[4] It will be shown qualitatively that using the linear resistive grid kernel estimator (RGKE) provides results comparable to an estimate formed using a Gaussian kernel. Results using the nonlinear RGKE will illustrate the ability of the resistive grid to detect discontinuities in the density. Finally, conclusions and possible applications will be discussed.

CONTINUOUS RGKE

The resistive network described in Mead[2] provides a means of computing the weighted average of many input signals or observations. The voltage at a node is determined by the weighted average of the inputs. Since the amplitude of the voltage due to a single input decreases exponentially with distance, signals that are farther away will carry less weight. A schematic of the one-dimensional resistive network is shown in Figure 1.* The equation for the general univariate kernel estimator of constant window width h is

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$$

where n is the number of data points, h is a smoothing parameter, and K denotes a kernel. For a one-dimensional continuous resistive network, the following equation can be used as the kernel centered at y

$$K_{RG}(y) = Ae^{-\frac{|x-y|}{L}}, \quad L = \frac{1}{\sqrt{RG}}$$

where L is the characteristic length, R is the resistance, G is the conductance, and A is a normalization constant. In this case, the characteristic length L can be identified with the window width h. The normalized voltage at each node of the resistive network described in Mead[2] determines the estimate of the density function.

Figure 2 shows a comparison of kernel estimates of a normal density using the continuous RGKE and Gaussian kernels for various window widths.[5] It can be seen from these plots that, as expected, the RGKE yields a density estimate similar to one using Gaussian kernels.

---

* All figures are included after the References.

Since the continuous RGKE is a bounded Borel function satisfying the following conditions

$$\int \left|K_{RG}(x)\right| dx < \infty$$

$$\int K_{RG}(x)dx = 1$$

$$\left|xK_{RG}(x)\right| \to 0, \quad |x| \to \infty$$

with $h \to 0$ and $nh \to \infty$, then the estimate $\hat{f}_{RG}(x) \to f(x)$

in probability as $n \to \infty$.[1]

DISCRETE LINEAR RGKE

When implementing the RGKE, it is necessary to consider quantized inputs.[2] The probability density estimate for the k-th node of a discrete network is given by

$$\hat{f}_{RG_k} = A\sum_{i=1}^{N} M_i \gamma^{|i-k|}$$

where M_i is the number of observations in bin i, A is a normalization constant, N is the number of nodes, and

$$\gamma \approx 1 + \frac{1}{2L^2} - \frac{1}{L}\sqrt{1 + \frac{1}{4L^2}}$$

The expression for γ becomes exact as the number of nodes in the discrete network approaches infinity. This equation is valid under the assumption that linear superposition holds.

To simulate the linear RGKE, 10,000 data points are drawn from a quantized normal density with zero mean and variance one, similar to that shown in Figure 3. The probability density function is then estimated only at the centers of the bins. Figure 4 illustrates the estimate from a discrete linear resistive grid with a characteristic length of 5.0. This shows that the RGKE can yield a smooth density estimate of the data.

A discontinuity is introduced in the data to evaluate the performance of the linear discrete RGKE. This is illustrated in Figure 5a, which clearly shows the edge in the data. The estimates from the RGKE for different values of L are shown in Figures 5b through 5d. As these plots show, by decreasing L the discontinuity can be detected. However, smoothness in the estimate is lost when the characteristic length is made small enough to detect discontinuities in the data. This fundamental tradeoff in the choice of the smoothing parameter L is an inherent characteristic of linear kernel estimators.

DISCRETE NONLINEAR RGKE

The above equations for continuous and linear discrete RGKE allow for a development of the theory with respect to kernel estimators. However, including the nonlinearities of the network in the formulation has some benefits. It can be shown that the following equations govern the voltage at the i-th node of a resistive grid network when linearity is not assumed:

$$V_i = \frac{GV_i^{in} + \frac{V_{i-1}}{R_{i-1}} + \frac{V_{i+1}}{R_i}}{G + \frac{1}{R_{i-1}} + \frac{1}{R_i}} \quad , \quad V_i^{in} = \beta M_i$$

with

$$R_i = \frac{R_0 \frac{V_{i+1} - V_i}{2}}{\tanh \frac{V_{i+1} - V_i}{2}}$$

where β is a scaling factor controlling the degree of nonlinearity, V_i^{in} is the fixed input voltage to the i-th node, and R_0 is the zero signal resistance value. Again, the estimate of the density function is given by the voltages at each node

$$\hat{f}_{NLRG_i} = \frac{V_i}{\sum_{i=1}^{N} V_i}$$

This set of coupled nonlinear equations is applied to the discontinuous data of the previous section. Again, a histogram of the data is shown in Figure 6a. Figures 6b-d illustrate the probability density estimates for different degrees of nonlinearity. A value of 0.2 for β yields results similar to the linear case; the curve is very smooth and no discontinuity is detected in the density. However, as more nonlinearity is allowed, the discontinuity in the density becomes apparent. With $\beta = 1.0$, the curve remains smooth while the discontinuity in the density function is clearly shown. Thus, the inherent nonlinearity in a RGKE can actually improve its probability density functional estimator qualities.

CONCLUSIONS AND APPLICATIONS

This study has shown qualitatively that the RGKE yields results comparable to probability density estimates derived using Gaussian kernels. The ability of the nonlinear RGKE to detect discontinuities in the density, while continuing to produce a smooth function, illustrates its usefulness as an estimator. Since the RGKE is suitable for hardware implementation, it is possible to apply it to problems in discriminant analysis and pattern classification when real-time responses are required.

REFERENCES

1. Silverman, B. W., *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, London, 1986.

2. Mead, Carver, *Analog VLSI and Neural Systems*, Addison-Wesley, 1989.

3. Specht, D. F., "Probabilistic Neural Networks," *Neural Networks*, Vol. 3, 1990, pp 109-118.

4. Lippmann, R. P., "Pattern Classification Using Neural Networks," *IEEE Communications Magazine*, Nov 89, pp 47-64.

5. Duda, R.O. and Hart, P., *Pattern Classification and Scene Analysis*, Wiley & Sons, New York, 1973.

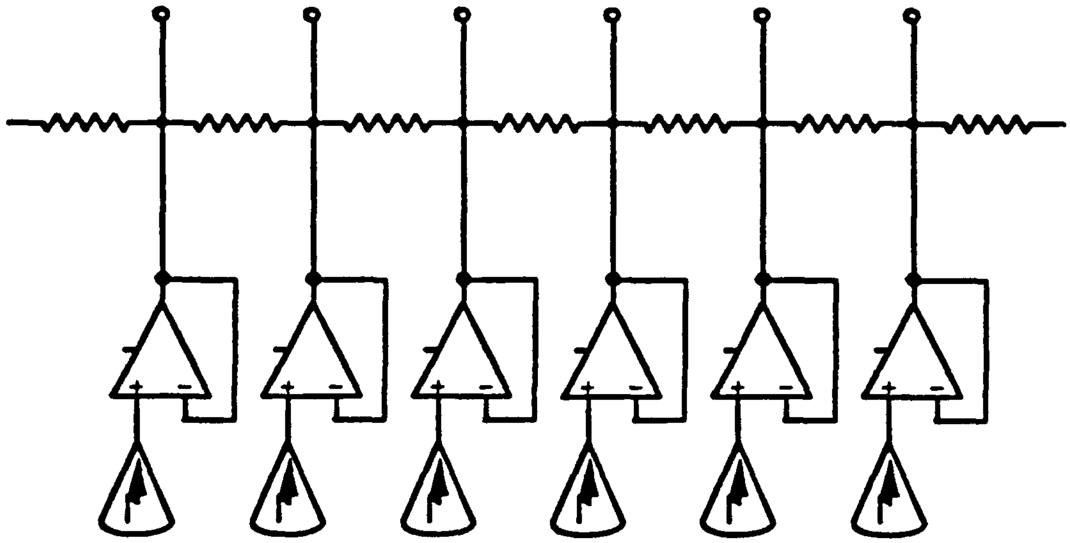

FIGURE 1. SCHEMATIC OF RESISTIVE GRID NETWORK

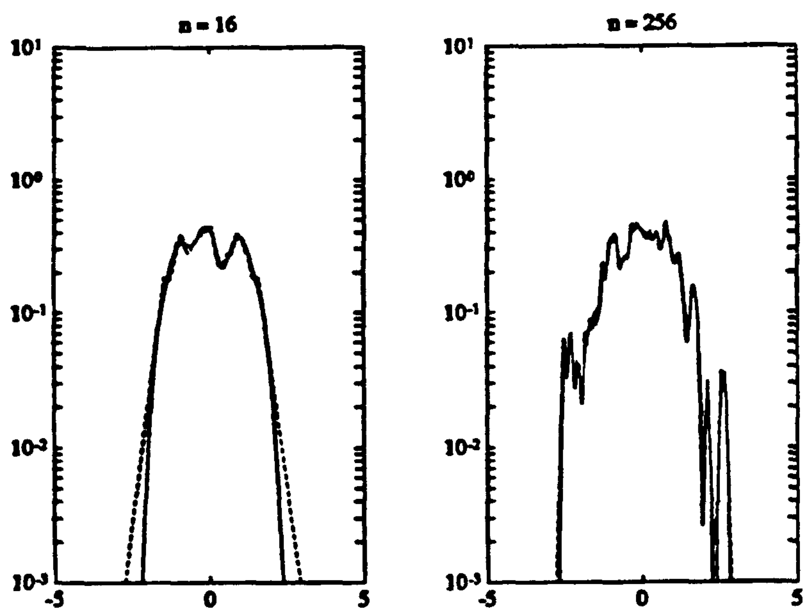

FIGURE 2. COMPARISON BETWEEN AN ESTIMATE USING GAUSSIAN KERNEL (SOLID) AND RESISTIVE GRID KERNEL (DOTTED)

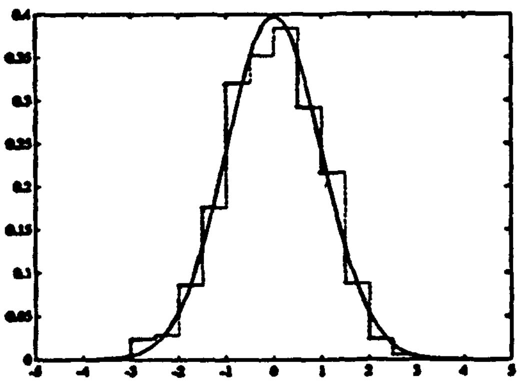

FIGURE 3. EXAMPLE OF DATA DRAWN FROM NORMAL DISTRIBUTION, WHICH HAS BEEN QUANTIZED FOR RESISTIVE GRID NETWORK

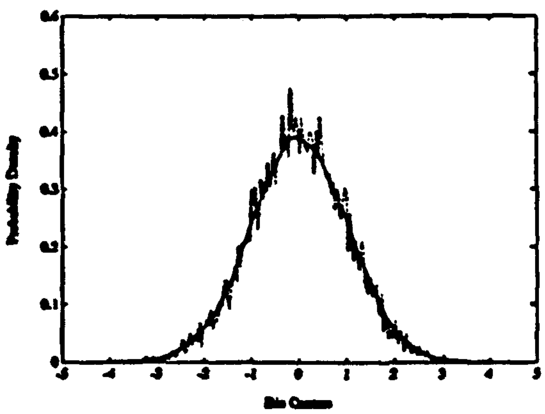

FIGURE 4. COMPARISON BETWEEN ESTIMATE USING HISTOGRAM (DOTTED) AND DISCRETE LINEAR RGKE (SOLID) WITH L=5.0

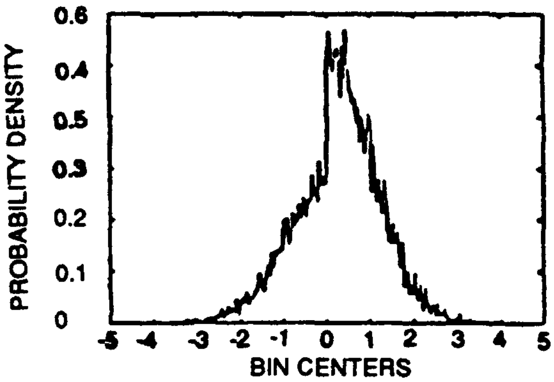

FIGURE 5A. ESTIMATE OF DENSITY USING HISTOGRAM

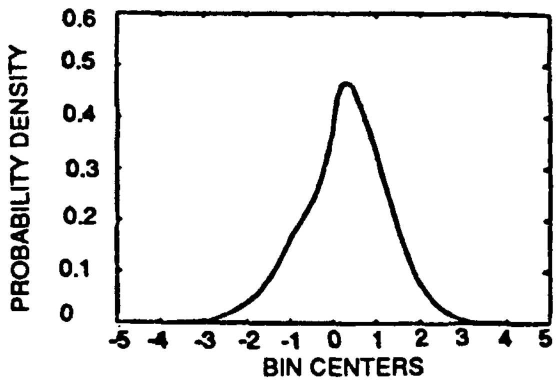

FIGURE 5B. LINEAR RGKE ESTIMATE WITH L=5.0

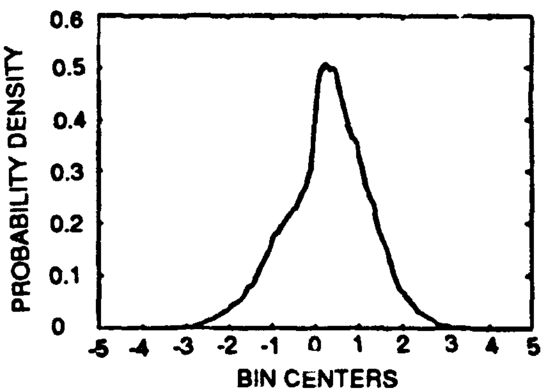

FIGURE 5C. LINEAR RGKE ESTIMATE WITH L=2.0

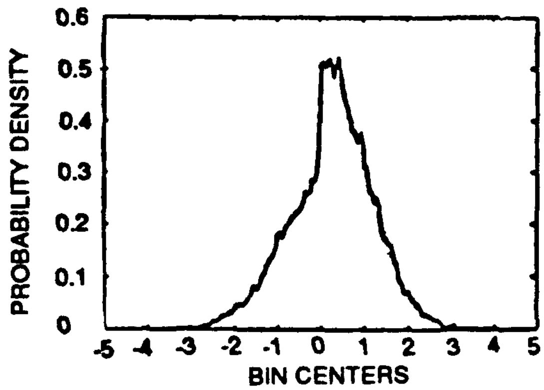

FIGURE 5D. LINEAR RGKE ESTIMATE WITH L=1.0

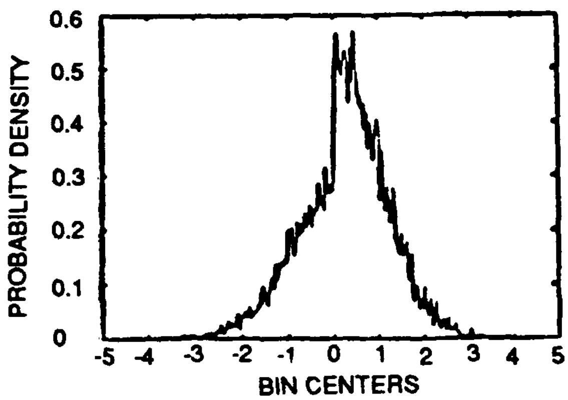

FIGURE 6A. ESTIMATE OF DENSITY USING HISTOGRAM

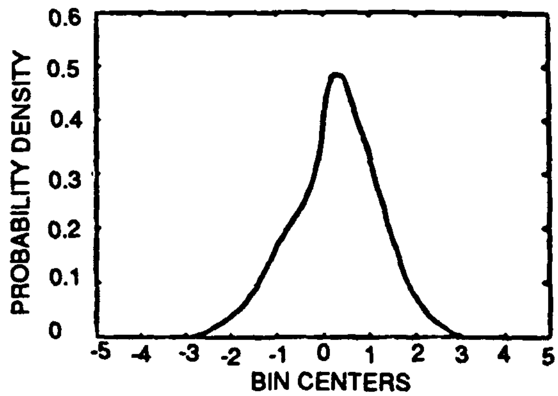

FIGURE 6B. NONLINEAR RGKE ESTIMATE WITH β=0.2

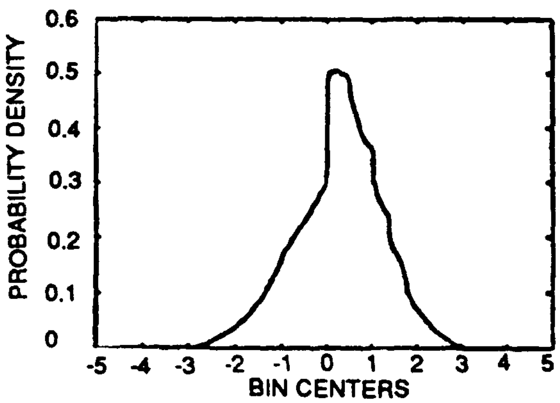

FIGURE 6C. NONLINEAR RGKE ESTIMATE WITH β=1.0

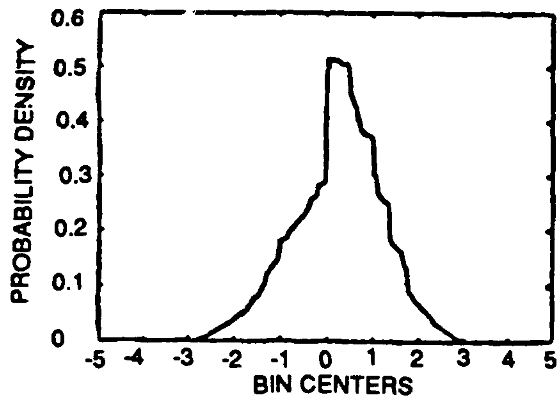

FIGURE 6D. NONLINEAR RGKE ESTIMATE WITH β=2.0

DISTRIBUTION (CONTINUED)

	COPIES
ATTN CODE 213 SIEGEL OFFICE OF NAVAL TECHNOLOGY 800 N QUINCY ST ARLINGTON VA 22217-5000	1
ATTN THOMAS MCKENNA OFFICE OF CHIEF OF NAVAL RESEARCH COGNITIVE AND NEURAL SCIENCE DIV 800 N QUINCY ST ARLINGTON VA 22217-5000	1
ATTN CLIFFORD LAU OFFICE OF CHIEF OF NAVAL RESEARCH ELECTRONICS DIV 800 N QUINCY ST ARLINGTON VA 22217-5000	1
ATTN DSO YOUN DEFENSE ADVANCED RESEARCH PROJECTS AGENCY 1400 WILSON BLVD ROSLYN VA 22201	1
ATTN PAUL WERBOS NATIONAL SCIENCE FOUNDATION ROOM 1151 WASHINGTON DC 20550	1
ATTN AL GORDON 013	1
DAVID MARCHETTE COMMANDER NAVAL OCEAN SYSTEMS CENTER SAN DIEGO CA 92152-5000	1
ATTN ROY STREIT OFFICER IN CHARGE NAVAL UNDERSEA WARFARE CENTER DETACHMENT NEW LONDON CT 06320	1
ATTN JIM MAAR R51 NATIONAL SECURITY AGENCY FT MEADE MD 20755-6000	1
DEFENSE TECHNICAL INFORMATION CENTER CAMERON STATION ALEXANDRIA VA 22304-6145	2

INTERNAL DISTRIBUTION:

		COPIES
C		1
D		1
D4		1
E231		3
E232		2
E261	FINK	1
E32	GIDEP	1
E42	HYNSON	1
F33	TURNER	1
F44	POWLAK	1
G11	LUCAS	1
G23	MALYEVAC	1
G32	HOLLAND	1
G42	FARSAIE	1
G42	DUMOULIN	1
G43	CAVARIS	1
H23	DRAPER	1
K		1
K10		5
K12		5
K12	ROGERS	5
K12	POSTON	5
K12	PRIEBE	5
K12	SOLKA	5
K13		1
K14		1
K44	GLASS	1
K44	REID	1
N415	MCCLINTOCK	1
N415	HARMON	1
R44	SZU	1
R44	TELFER	1
U23	BARAN	1

REPORT DOCUMENTATION PAGE

Form Approved
OBM No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
June 1992

3. REPORT TYPE AND DATES COVERED

4. TITLE AND SUBTITLE
Resistive Grid Kernel Estimator (RGKE)

5. FUNDING NUMBERS

6. AUTHOR(S)
Wendy Poston, George Rogers, Carey Priebe, and Jeffrey Solka

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Surface Warfare Center Dahlgren Division (K12)
Dahlgren, Virginia 22448-5000

8. PERFORMING ORGANIZATION REPORT NUMBER
NSWCDD/TR-92/241

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The ability to estimate a probability density function from random data has applications in discriminant analysis and pattern recognition problems. A resistive grid kernel estimator (RGKE) is described that is suitable for hardware implementation. The one-dimensional linear RGKE is compared to a kernel estimate using Gaussian kernels, and simulations are presented using both continuous and quantized data. The nonlinear form of the RGKE is shown to have desirable properties, such as the ability to detect discontinuities in the density function.

14. SUBJECT TERMS
Neural Networks, Resistive Grid Kernel Estimator, Resistive Grid Network

15. NUMBER OF PAGES
15

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102